۱۰۱)

با توجه به اینکه زیر رادیکال زوج باید نامنفی باشد $\longrightarrow$ تعیین علامت $\longrightarrow x f^{-1}_{(x)} \geq 0$

ریشه: $f^{-1}_{(x)} = 0 \longrightarrow (x,0) \in f^{-1} \longrightarrow (0,x) \in f \longrightarrow (x=3)$

		0		3	
x	$-$	0	$+$		$+$
$f^{-1}_{(x)}$	$+$		$+$	0	$-$
$x f^{-1}_{(x)}$	$-$	0	$+$	0	$-$

$$\longrightarrow D_{g(x)} = [0,3]$$

---

۱۰۲)

بالای محور xها $\longrightarrow$ $\Delta < 0$ ، ضریب x^2: $a > 0$

$$\Delta = b^2 - 4ac \longrightarrow \Delta = (2\sqrt{6})^2 - 4(1-a)(-a) < 0 \longrightarrow 24 + 4a - 4a^2 < 0 \longrightarrow$$

$$a^2 - a - 6 > 0 \longrightarrow (a-3)(a+2) > 0 \longrightarrow$$

		-2		3	
	$+$	0	$-$	0	$+$

$\boxed{(a<-2) \cup (a>3)}$ I)

$(1-a) > 0 \longrightarrow \boxed{a<1}$ II)

$\mathrm{I} \cap \mathrm{II}) \longrightarrow \boxed{a<-2}$

---

۱۰۳)

چون از 2 نقطه داده شده 2 نقطه در معادله صدق می‌کند.

$$11 = a + \log_2 (15+b)^2 \longrightarrow 11 = a + 2\log_2 (15+b)$$

$$15 = a + \log_2 (63+b)^2 \longrightarrow 15 = a + 2\log_2 (63+b)$$

$$\xrightarrow{\text{حل دستگاه}} 4 = 2\log_2 \frac{63+b}{15+b}$$

$$\log_2 \frac{63+b}{15+b} = 2 \longrightarrow \boxed{b=1} \xrightarrow[\text{هر یک معادله}]{\text{قرار دادن}} \boxed{a=3}$$

104) با توجه به شکل دوره تناوب اصلی تابع برابر $T = \frac{2\pi}{3}$ می‌باشد.

$$\frac{2\pi}{|m|} = \frac{2\pi}{3} \rightarrow \boxed{m = \pm 3} \rightarrow$$

قابل قبول $\boxed{m=3}$

$$y = 1 - \sin 3\left(\frac{7\pi}{6}\right) \rightarrow y = 1 - \sin\left(\frac{7\pi}{2}\right) = 1-(-1) = \underline{\underline{2}}$$

---

105) چون 2 نقطه تقاطع می‌باشد ← $f(x) = g(x)$

$$g(x) = \frac{1}{4^x} + \frac{3}{2} \;,\; f(x) = 4^x$$

$$\frac{1}{4^x} + \frac{3}{2} = 4^x \xrightarrow{4^x = t} \frac{1}{t} + \frac{3}{2} = t \xrightarrow{\times 2t} 2 + 3t = 2t^2 \rightarrow$$

$$2t^2 - 3t - 2 = 0 \rightarrow \begin{cases} t = 2 \longrightarrow 4^x = 2 \rightarrow x = \frac{1}{2} \rightarrow \boxed{y = 2} \\ t = -\frac{1}{2} \quad \text{غ.ق.ق} \end{cases}$$

$A\left(\frac{1}{2}, 2\right) \rightarrow$ نقطه‌ی برخورد

$$\sqrt{\left(\frac{1}{2} - \left(-\frac{1}{2}\right)\right)^2 + (2-1)^2} = \sqrt{1+1} = \underline{\underline{\sqrt{2}}}$$

---

106)

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3(\alpha\beta)(\alpha + \beta)$$

$$\alpha^3 + \beta^3 = \left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right) \rightarrow \alpha^3 + \beta^3 = \left(\frac{1}{2}\right)^3 - 3(-1)\left(\frac{1}{2}\right) \rightarrow$$

$$\rightarrow \alpha^3 + \beta^3 = \frac{1}{8} + \frac{3}{2} \rightarrow \alpha^3 + \beta^3 = \frac{1+12}{8} \Rightarrow \alpha^3 + \beta^3 = \frac{13}{8}$$

$$\frac{+m}{8} = \frac{13}{8} \rightarrow \boxed{m = 13}$$

107)

$D_{gof} = \{x \in D_f \mid f(x) \in D_g\}$ $\quad \boxed{D_f = \mathbb{R}}$

$D_g : x - x^2 \geq 0 \rightarrow \boxed{D_g = 0 \leq x \leq 1}$

$D_{gof} = \left\{ x \in \mathbb{R} \;\middle|\; 0 \leq \dfrac{1-x^2}{1+x^2} \leq 1 \right\}$

$\dfrac{1-x^2}{1+x^2} \geq 0 \longrightarrow -1 \leq x \leq 1 \quad (I)$

$\boxed{I \cap II \longrightarrow [-1,1]}$

$\dfrac{1-x^2}{1+x^2} \leq 1 \longrightarrow \dfrac{1-x^2}{1+x^2} - 1 \leq 0 \rightarrow \dfrac{1-x^2-1-x^2}{1+x^2} \leq 0 \rightarrow \dfrac{-2x^2}{1+x^2} \leq 0 \xrightarrow{(II)}$ همواره برقرار باشد

108)

$\cos(\frac{\pi}{2} - \alpha) = \sin\alpha$

$\cos\left(\frac{\pi}{2} - 2\tan^{-1}(-\frac{1}{2})\right) = \sin\left(2\tan^{-1}(-\frac{1}{2})\right) \xrightarrow{\tan^{-1}(-\frac{1}{2}) = \alpha} \sin(2\alpha)$

$\rightarrow \sin 2\alpha = \dfrac{2\tan\alpha}{1+\tan^2\alpha} \rightarrow \sin 2\alpha = \dfrac{2 \times -\frac{1}{2}}{1+\frac{1}{4}} \Rightarrow \sin 2\alpha = \dfrac{-1}{\frac{5}{4}}$

$\rightarrow \boxed{\sin 2\alpha = -\dfrac{4}{5}}$

109)

$4\cos 40 - \dfrac{1}{\cos 20} = \dfrac{4\cos 40 \cos 20 - 1}{\cos 20} = \dfrac{4\left(\frac{1}{2}(\cos 60 + \cos 20)\right) - 1}{\cos 20} = \rightarrow$

$\rightarrow \dfrac{2(\cos 60 + \cos 20) - 1}{\cos 20} = \dfrac{\cancel{1} + 2\cos 20 \cancel{-1}}{\cos 20} = \dfrac{2\cos 20}{\cos 20} = 2$

۱۰) ب

$$\sin x + \sin 2x + \sin 3x = 0$$

$$2\sin 2x \cos x + \sin 2x = 0 \longrightarrow \sin 2x \left[2\cos x + 1\right] = 0$$

$$\sin 2x = 0 \longrightarrow 2x = k\pi \longrightarrow x = \frac{k\pi}{2} \longrightarrow$$ غ.ق.ق

ب) طبق قید مسئله

$$\cos x = -\frac{1}{2} \longrightarrow \cos x = \cos\left(\frac{2\pi}{3}\right) \longrightarrow \boxed{x = 2k\pi \pm \frac{2\pi}{3}}$$

---

۱۱)

$$\lim_{x \to 0^+} \frac{1 - \cos\sqrt{x}}{x} \times \frac{1 + \cos\sqrt{x}}{\underbrace{1 + \cos\sqrt{x}}_{2}} = \lim_{x \to 0^+} \frac{1 - \cos^2\sqrt{x}}{2x} = \lim_{x \to 0^+} \frac{\sin^2\sqrt{x}}{2x}$$

$$= \lim_{x \to 0^+} \frac{(\sqrt{x})^2}{2x} = \lim \frac{x}{2x} = \frac{1}{2}$$

$$\tan\big(\underbrace{\tfrac{\pi}{6}+\sin^{-1}\sqrt{x}}_{v}\big)$$ ۱۱۲)

$$\frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x}} * \left(1+\tan^2\left(\frac{\pi}{6}+\sin^{-1}\sqrt{x}\right)\right) \xrightarrow{x=\frac{1}{4}}$$

$$\rightarrow f'\left(\tfrac{1}{4}\right) = \frac{\frac{1}{2\times\frac{1}{2}}}{\sqrt{\frac{3}{4}}} * \left(1+\tan^2\left(\frac{\pi}{6}+\underbrace{\sin^{-1}\tfrac{1}{2}}_{\frac{\pi}{6}}\right)\right)$$

$$f'\left(\tfrac{1}{4}\right) = \frac{2}{\sqrt{3}} \times \left(1+(\sqrt{3})^2\right) = \underline{\underline{\frac{8}{\sqrt{3}}}}$$

113)

$b_n = n\ln(n+1)$

$a_n = n\ln(n)$ $\longrightarrow b_n - a_n = n\ln(n+1) - n\ln(n)$

$\longrightarrow b_n - a_n = n\left(\ln(n+1) - \ln(n)\right) \Rightarrow b_n - a_n = n\left(\ln\frac{(n+1)}{n}\right)$

$\longrightarrow b_n - a_n = \ln\left(\frac{n+1}{n}\right)^n$ I)

ابهام به صورت 1^∞ می باشد

$\lim_{n\to+\infty}\left(\frac{n+1}{n}\right)^n = e^{\lim_{n\to+\infty}\left(\frac{n+1}{n}-1\right)\times n} \rightarrow = e^1$

دنباله همگرا به 1 می باشد

$\xrightarrow{I} \ln(e) = 1$

---

114) باید حد چپ و حد راست تابع را در نقطه $x=0$ بیابیم.

$\begin{cases} \sin x < x & x>0 \\ x < \sin x & x<0 \end{cases}$ طرفین ÷ x $\longrightarrow \begin{cases} \frac{\sin x}{x} < 1 & x>0 \\ \frac{\sin x}{x} < 1 & x<0 \end{cases}$

$\lim_{x\to 0}\left[\frac{\sin x}{x}\right] = 0 \longrightarrow 0\times\cos 4x = 0$

پس برای اینکه تابع در $x=0$ پیوسته باشد $a=0$ می باشد

115) با استفاده از هم ارزی توانی

$$y=\sqrt[3]{x^3-x^2} \longrightarrow \sim \sqrt[3]{1}\left(x+\frac{-1}{3}\right)=x-\frac{1}{3}$$

$$\sqrt[3]{x^3-x^2}=x-\frac{1}{3} \xrightarrow{\text{طرفین توان } 3} x^3-x^2=x^3-\frac{1}{27}+3\left(\frac{1}{9}\right)(x)-3x^2\left(\frac{1}{3}\right)$$

$$\longrightarrow \cancel{x^3}-\cancel{x^2}=\cancel{x^3}-\cancel{x^2}+\frac{x}{3}-\frac{1}{27} \longrightarrow \frac{x}{3}=\frac{1}{27} \longrightarrow \boxed{x=\frac{1}{9}}$$

---

116) با توجه به اینکه معادله درجه سوم در $\mathbb{R}$ پیوسته می‌باشد با استفاده از قضیه بولتزانو

$$f\left(\frac{1}{3}\right)=3\left(\frac{1}{27}\right)+a\left(\frac{1}{3}\right)-1 \longrightarrow f\left(\frac{1}{3}\right)=\frac{1}{9}+\frac{a}{3}-1 \Rightarrow f\left(\frac{1}{3}\right)=\frac{3a-8}{9}$$

$$f\left(\frac{1}{2}\right)=3\left(\frac{1}{8}\right)+a\left(\frac{1}{2}\right)-1 \longrightarrow f\left(\frac{1}{2}\right)=\frac{3}{8}+\frac{a}{2}-1 \longrightarrow f\left(\frac{1}{2}\right)=\frac{4a-5}{8}$$

$$f\left(\frac{1}{3}\right)f\left(\frac{1}{2}\right)<0 \longrightarrow \left(\frac{3a-8}{9}\right)\left(\frac{4a-5}{8}\right)<0 \longrightarrow$$ پس الف

	$\frac{5}{4}$		$\frac{8}{3}$	
$+$	0	$-$	0	$+$

$$\longrightarrow \left(\frac{5}{4},\frac{8}{3}\right)$$

117) شیب ساده‌ترین خط مماس در نقطه‌ی زاویه‌دار 1 و -1 می‌باشد.

$$f_{+}(1)=\frac{(x-1)}{\sqrt{x^2+3}} \longrightarrow f'_{+}(1)=\frac{(1)(\sqrt{x^2+3})-\frac{2x}{2\sqrt{x^2+3}}\times(x-1)}{x^2+3}=\frac{2}{4}=\frac{1}{2}$$

$$f_{-}(1)=-\frac{(x-1)}{\sqrt{x^2+3}} \Rightarrow f'_{-}(1)=-\frac{(1)(\sqrt{x^2+3})-\frac{2x}{2\sqrt{x^2+3}}(x-1)}{x^2+3}=-\frac{2}{4}=-\frac{1}{2}$$

$$\tan^{-1}\theta=\left|\frac{\frac{1}{2}-(-\frac{1}{2})}{1+(\frac{1}{2})(-\frac{1}{2})}\right| \longrightarrow \tan\theta=\frac{1}{\frac{3}{4}} \longrightarrow \boxed{\tan\theta=\frac{4}{3}}$$

118) با توجه به داده‌ی مسئله

$$\lim_{h\to 0}\frac{f(-2+h)+3}{h}=\frac{1}{2} \longrightarrow \begin{cases} f'(-2)=\frac{1}{2} \\ f(-2)=-3 \end{cases}$$

$$(x^2f(x))'=2xf(x)+x^2f'(x) \xrightarrow{x=-2} (-4)f(-2)+(-2)^2f'(-2) \longrightarrow$$

$$\longrightarrow (-4)\times(-3)+(4)(\frac{1}{2})=12+2=\underline{\underline{14}}$$

دانلود از سایت ریاضی سرا

www.riazisara.ir

(۱۱

$(1,a)\in f^{-1} \to (a,1)\in f$

$\to (1,0)\in f^{-1} ، (0,1)\in f$

$1 = x + e^{2x} \to \boxed{x=0}$

$f^{-1\prime}(1) = \dfrac{1}{f'(0)} \to \boxed{f^{-1\prime}(1) = \dfrac{1}{3}}$

$f'(x) = 1+2e^{2x} \to f'(0) = 1+2e^{0} \to f'(0) = 3$

$y-y_0 = m(x-x_0) \to y-0 = \frac{1}{3}(x-1) \to y = \frac{1}{3}(x-1)$

$\to 3y = x-1 \to \boxed{3y-x=-1}$

(۱۲

$x^2y - \ln(2x-y) = 12 \to x^2y - \ln(2x-y) - 12 = 0$

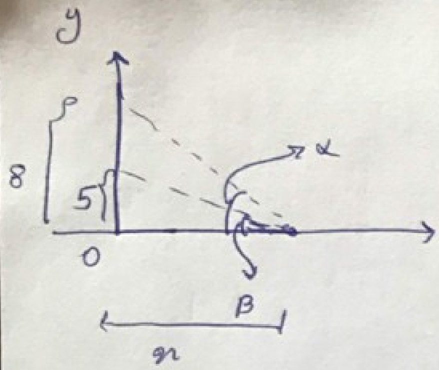
$f'(x,y) = -\dfrac{2xy - \dfrac{2}{2x-y}}{x^2 - \dfrac{-1}{2x-y}} \xrightarrow{(2,3)} f'(x,y) = -\dfrac{12-\dfrac{2}{1}}{4-\dfrac{-1}{1}} = \dfrac{10}{5}$

$\to f'(x,y) = -2 \xrightarrow{\text{شیب خط عمود}} \boxed{m=\frac{1}{2}}$

$y-y_0 = m(x-x_0) \to y-3 = \frac{1}{2}(x-2) \xrightarrow{y=0} -3 = \frac{1}{2}(x-2) \to$

$-6 = x-2 \to \boxed{x=-4}$

121



$$\tan\alpha = \frac{8}{x} \longrightarrow \boxed{\alpha = \tan^{-1}\frac{8}{x}}$$

$$\tan\beta = \frac{5}{x} \longrightarrow \boxed{\beta = \tan^{-1}\frac{5}{x}}$$

$$\alpha - \beta \longrightarrow max \rightarrow \tan^{-1}\frac{8}{x} - \tan^{-1}\frac{5}{x} \longrightarrow max$$

$$\frac{-\frac{8}{x^2}}{1+\frac{64}{x^2}} - \frac{-\frac{5}{x^2}}{1+\frac{25}{x^2}} = 0 \rightarrow \frac{\cancel{-}\frac{8}{\cancel{x^2}}}{\frac{x^2+64}{\cancel{x^2}}} = \frac{\cancel{-}\frac{5}{\cancel{x^2}}}{\frac{x^2+25}{\cancel{x^2}}}$$

$$8x^2+200 = 5x^2+320 \rightarrow 3x^2 = 120 \rightarrow x^2 = 40 \rightarrow \boxed{x = 2\sqrt{10}}$$

122

$$f(x) = \sin^2 x - 2\sin x \longrightarrow f'(x) = 2\sin x\cos x - 2\cos x$$

$$f'(x) = 2\cos x(\underbrace{\sin x - 1}_{\text{نامثبت}}) \geq 0 \longrightarrow \boxed{\cos x \leq 0} \rightarrow \mathrm{I}$$

$$f''(x) = 2(\underbrace{\cos^2 x - \sin^2 x}_{1-2\sin^2 x}) + 2\sin x \longrightarrow f''(x) = 2(1-2\sin^2 x) + 2\sin x$$

$$f''(x) = -4\sin^2 x + 2\sin x + 2 \longrightarrow f''(x) = -2(2\sin^2 x - \sin x - 1) \geq 0$$

$$f''(x) = -2(\underbrace{\sin x - 1}_{\text{نامثبت}})(2\sin x + 1) \geq 0 \longrightarrow \sin x \leq -\frac{1}{2} \rightarrow \mathrm{II}$$

آنگاه بازه ۱ و ۲ مشترک
I و II اشتراک می‌شود

123)

$$f(c)=\frac{\int_1^9 \frac{1}{x}dx}{9-1} \longrightarrow f(c)=\frac{\ln x\Big|_1^9}{9-1} \longrightarrow f(c)=\frac{\ln 9-\cancel{\ln 1}}{8}$$

$$\longrightarrow \frac{1}{c}=\frac{\ln 9}{8} \longrightarrow c=\frac{8}{\ln 9} \longrightarrow c=\frac{8}{2\ln 3} \longrightarrow \boxed{c=\frac{4}{\ln 3}}$$

---

124)

$$\int_0^4 \sqrt{(x^2-2x)^2}\,dx=\int_0^4 |x^2-2x|\,dx=$$

* باید ببینیم ریشه‌های داخل قدرمطلق درون محدوده انتگرال را بازه‌بندی کنیم *

$$\int_0^2 (2x-x^2)\,dx\,dx+\int_2^4 (x^2-2x)\,dx \longrightarrow$$

$$\int_0^2 2x\,dx-\int_0^2 x^2\,dx+\int_2^4 x^2\,dx-\int_2^4 2x\,dx \longrightarrow$$

$$x^2\Big|_0^2-\frac{x^3}{3}\Big|_0^2+\frac{x^3}{3}\Big|_2^4-x^2\Big|_2^4 \longrightarrow$$

$$(4-0)-\left(\frac{8}{3}-0\right)+\left(\frac{64}{3}-\frac{8}{3}\right)-(16-4) \longrightarrow$$

$$4-\frac{8}{3}+\frac{56}{3}-12=-8-\frac{8}{3}+\frac{56}{3}=\frac{24}{3}=8$$